Ph.D. Qualify Examination 2019
Theory of Computation

- This examination is closed books.
- Please turn off your cell phones.
- Remember that there are 2 pages of the qualify examination.
- Answer all questions as possible. You may have a partial score if you answer the correct direction.

1. Deterministic Finite Acceptor (DFA) (10 pts)

   Find a dfa for the following language on $\Sigma = \{0, 1\}$:
   All strings containing 000 but not 0000.

2. Nondeterministic Finite Acceptor (NFA) (10 pts)

   Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.

3. Find the minimum dfa's for the following language and prove that the result is minimal. (10 pts)
   $L = \{a^n : n \geq 0, n \neq 2\}$.

4. Determine whether or not the following language on $\Sigma = \{a\}$ is regular: (10 pts)
   $L = \{a^n : n = k^3$ for some k $\geq 0\}$.

5. Show that the following grammar is ambiguous. (10 pts)
   $S \to AB|aaB$,

   $A \to a|Aa$,

   $B \to b$.

6. In the derivation of using pumping lemma to prove a language L is not regular, we will give an assumption "A DFA M with the number of states $|M|$ exists for L". Can you replace the assumption with "An NFA M' with the number of states $|M'|$"? Please justify your answer. (15 pts)

7. Construct a nondeterministic pushdown automata that accepts the following language on $\Sigma = \{a, b, c\}$: (10 pts)

   $L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$.

8. Suppose r_1 and r_2 are regular expressions, and $L(r_1)$ and $L(r_2)$ are regular languages, correspondingly. Prove that $L(r_1 + r_2)$ is also a regular language (15 pts).

   hint: Note that $L(r_1 + r_2) = L(r_1) \cup L(r_2)$. You can consider the solution by using NFA.

9. Convert the NFA in Figure 1 into an equivalent DFA. (10 pts)

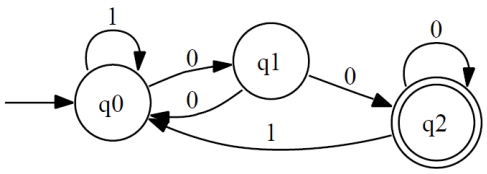

Figure 1: The NFA case.